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MATHEMATICAL MODEL OF HOT-CAST MOLDING OF CERAMIC

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The results of mathematical modeling of hot-cast molding of ceramic are presented. The computational data describe the changes of the aggregate state of thermoplastic slip as it cools in the molding cavity. The casting properties giving uniform properties of the thermoplastic slip during hot-cast molding of ceramic are obtained.

Key words: mathematical model, thermoplastic slip, molding process, heat transfer.

Hot casting of thermoplastic slip is currently the main method of molding complex parts as well as tubular and rod-shaped parts [1]. The essence of this fabrication method is to impart to liquid thermoplastic slip the configuration of the part by filling a molding cavity under pressure and then fixing the acquired shape by changing the aggregate state of the medium as it cools.

The casting system consists of a multiphase medium where a thermoplastic binder with complex composition and coagulation properties is used as the dispersion phase [2, 3]. The rheological properties of the slip are determined by the ratio between the kinetically free and bound dispersion phase. Beryllium oxide is a promising material for a number of special applications in technology. However, its unique thermal conductivity at the molding stage gives casting systems an elevated stiffness, which makes it difficult to control structure formation during slip motion [2].

The method of hot casting includes the following stages [1, 2]: slip motion and heat transfer in the liquid state; slip motion and heat transfer taking account of solidification; cast motion and heat transfer in the solid-plastic state. At the stage when the molding cavity is filled and held under pressure it is most important to ensure the maximum destruction of the structure in order to obtain a uniform suspension [2]. During solidification it is necessary to attain minimum friction against the wall of the molding cavity and maximum plasticity without repeated destruction of the newly formed cast structure.

The hydrodynamics of thermoplastic slip during casting should be viewed as a physical process of deformation and flow. The slip flow retains its configuration after exiting the feeder. Experiments have established that in the range of possible casting rates the flow of thermoplastic slip in a casting mold is laminar [2]. Slip enters the casting mold at temperature 75 – 85°C and cools in the mold to 30 – 45°C, at which the cast can be extracted from the mold without being distorted [2].

The development and implementation of the method of hot casting have been largely empirical. A detailed analysis of the hydrodynamics and heat transfer will make it possible to approach on a more substantiated basis the technology of the slip casting regime.

We shall examine the results of a mathematical model of the process of hot-cast molding of beryllium oxide ceramic.

RHEOLOGICAL MODEL OF THERMOPLASTIC SLIP

It is assumed that thermoplastic slip (high-viscosity suspension) is a two-phase disperse system, where the solid mineral phase is a beryllium oxide powder while the liquid phase is an organic binder [2]. The organic binder consists of three components: paraffin, bees wax, and oleic acid in the mass ratio (wt.%) 82 : 15 : 3. Thermoplastic slip with thixotropic flow with shear rates in the range 0.005 – 1200 $\sec^{-1}$ can be treated as a Shvedov–Bingham viscoplastic liquid [4]:

$$\tau = \tau_0 + \mu \frac{\partial u}{\partial r}, \tag{1}$$

where τ is the shear stress, Pa; τ_0 is the maximum shear stress, Pa; μ is the plastic viscosity, Pa · sec; and, $\partial u/\partial r$ is the velocity gradient, $\sec^{-1}$.

The plastic viscosity μ and the maximum shear stress τ_0 of the slip depend on the temperature t. The experimental

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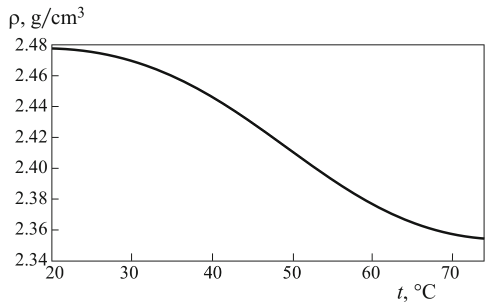

Fig. 1. Variation of the thermoplastic slip density versus temperature.

data of [2] with relative mass content of the binder $\omega = 0.10$ are described by the empirical relations

$$\mu = 1.17 \times 10^{9} \exp(-0.28t) + 49.4;$$

$$\tau_0 = 2.61 \times 10^{10} \exp(-0.339t) + 22.34.$$

The density of thermoplastic slip is determined by the beryllium oxide powder and binder concentrations

$$\rho = \frac{\rho_{\text{BeO}}\,\rho_{\text{св}}}{(1-\omega)\rho_{\text{св}} + \omega\rho_{\text{BeO}}},$$

where ρ_{BeO} is the density of beryllium oxide, g/cm^3; ρ_b is the binder density; and, ω is the relative mass content of the binder as a fraction of unity.

The binder density depends on the temperature and is determined by the empirical relation

$$\rho_b(t) = 0.83 + 0.062\cos[\pi(0.035t - 5.15)].$$

The beryllium oxide density is $\rho_{\text{BeO}} = 3.047$ g/cm^3. The binder density in the temperature range from 75 to 40°C varies within the limits $\rho_b = 0.773 - 0.9$ g/cm^3 and, accordingly, on solidification the thermoplastic slip density ρ increases from 2.355 to 2.460 g/cm^3 (Fig. 1).

Experimental data [2] show the empirical temperature dependences of the thermal conductivity λ and specific heat c_p of thermoplastic slip to be

$$\lambda = 1.6 + 4.8\exp(-0.017t);$$

$$c_p = 1000\exp(t/290).$$

In summary, the rheological properties of thermoplastic slip consisting of beryllium oxide are complicated functions of temperature and an aggregate change of the liquid suspension into a solid-plastic body occurs during casting.

MATHEMATICAL MODEL

The subject of study is the motion and heat transfer of thermoplastic slip in an annular cavity (Fig. 2). A hot liquid slip with initial temperature $t_0 = 75$°C flows into an annular cavity with thickness $(r_2 - r_1)$ and length L. According to the experimental data [2] the maximum initial velocity does not exceed $u_0 = 2$ mm/min. The slip mass cools and solidifies as it moves, acquiring shape at the exit from the molding cavity. A distinguishing feature of beryllium oxide slip is high thermal conductivity, but the Prandtl number $\Pr = \mu c_p/\lambda$ is much greater than 1 because of the viscosity of the thermoplastic slip. The wall of the molding cavity is cooled by water circulating in an annular case. The cooling zone is divided into three parts: the temperature $t_1 = 73$°C in the first part, $t_2 = 59$°C in the second, and $t_3 = 45$°C in the third. The total length of the cavity is $L = 0.108$ m and the lengths of the hot, warm, and cold parts are, respectively, $L_1 = 0.022$ m, $L_2 = 0.045$ m, and $L_3 = 0.041$ m.

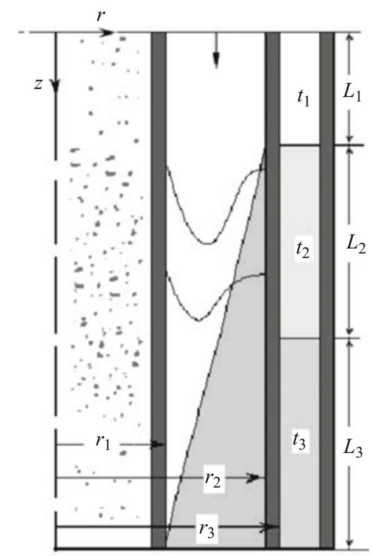

Fig. 2. Flow in a coaxial cavity: r_1, r_2) inner and outer radii of the annular channel; r_3) radius of the outer wall of the channel; L_1, L_2, L_3) lengths of the hot, warm, and cold loops; t_1, t_2, t_3) temperatures of the hot, warm, and cold loops, respectively.

The intensity of heat transfer between the hot slip and cooling liquid depends on the flow regime, thermophysical characteristics of the slip, temperature of the cooling liquid, material of the tube wall, and geometric dimensions of the case. In the course of the motion the rheological properties change because the liquid slip solidifies. Heat is released on the transition surface of the aggregate state of the slip. The slip mass in stationary motion in an annular cavity is cooled by the water laving the tube on the outside. A characteristic feature of solidification in the cold zone of the molding cavity is that the temperature t_i of the outer wall of the annular channel will be lower than the temperature t_w of the inner wall with radius r_1; this imparts nonuniformity to the temperature profile as well as to the rheological properties of the extruded slip. Solidification starts on the outer wall, while the slip will be in a liquid state at the inner wall of the annular cavity. As a result, liquid slip may be added in order to com-

pensate the interior shrinkage at the stage of solidification of the cast in the cooled zone of the cavity.

The problem is studied in a cylindrical coordinate system with axes z and r, directed parallel and radially to the flow, respectively. The slip motion is stationary in the direction of the casting velocity, and it can be studied using a system of equations of hydrodynamics, closed by the Bingham model of a non-Newtonian liquid. The thickness of the annular cavity is much smaller than the length, so that the narrow-channel approximation can be used to study the motion and heat transfer of the slip [4]:

$$\rho u\frac{\partial u}{\partial z}+\rho v\frac{\partial u}{\partial r}=-\frac{dp}{dz}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right)+$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tau_0)+\rho g; \tag{2}$$

$$\frac{\partial \rho u}{\partial z}+\frac{1}{r}\frac{\partial \rho v}{\partial r}=0; \tag{3}$$

$$\rho u c_p\frac{\partial t}{\partial z}+\rho v c_p\frac{\partial t}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left(\lambda r\frac{\partial t}{\partial r}\right)+H_k\frac{\partial \rho}{\partial s}+\mu\left(\frac{\partial u}{\partial r}\right)^2, \tag{4}$$

where z, r — axial and radial coordinates; u, v — components of the velocity vector; s — time; p, ρ, t, τ_0, c_p, μ, and λ — pressure, density, temperature, maximum shear stress, specific heat, viscosity, and thermal conductivity of the slip.

The heat of solidification H_k of the beryllium oxide slip is found from the experimental data and equals $H_k = 7800$ J/kg [5]. The rheological properties of the slip are expressed by empirical relations.

The pressure gradient in the equation of motion is determined from the conservation of the mass flow:

$$2\pi\int_{r_1}^{r_2}\rho u r dr=\pi(r_2^2-r_1^2)\rho_0 u_0. \tag{5}$$

The velocity and temperature distributions at the entrance into the annular cavity are assumed to be constant over the cross section of the channel and, correspondingly, all thermophysical properties of the slip are constant:

$$\text{for } z=0\text{: } u=u_0,\ v=0,\ t=t_0. \tag{6}$$

Adiabaticity is assumed on the inner wall of the cavity:

$$\text{for } z>0\text{: } r=r_1,\ \partial t/\partial r=0. \tag{7}$$

The conditions of impermeability and slippage of the slip are imposed on the walls of the annular layer:

$$\text{for } z>0,\ r=r_i,\ v=0,\ ,\ i=1,2. \tag{8}$$

Heat transfer on the outer wall occurs in accordance with the values of the temperature in the cooling loops of the cavity. Let t_1, t_2, and t_3 be the temperature of the water in the hot, warm, and cold loops, respectively. The boundary conditions for the temperature on the outer wall are:

$$\text{for } 0\le z\pi l_1,\ r=r_2\text{: } -\lambda\frac{\partial t}{\partial r}=k(t-t_1); \tag{9}$$

$$\text{for } l_1\le z\pi l_2,\ r=r_2\text{: } -\lambda\frac{\partial t}{\partial r}=k(t-t_2); \tag{10}$$

$$\text{for } l_2\le z\pi l_3,\ r=r_2\text{: } -\lambda\frac{\partial t}{\partial r}=k(t-t_3). \tag{11}$$

The coefficient of heat transfer k is calculated in the standard manner:

$$\frac{1}{kd_3}=\frac{1}{\alpha_2 d_2}+\frac{1}{\alpha_3 d_3}+\frac{1}{2\lambda_w \ln(d_3/d_2)},$$

where d_1, d_2 — inner and outer diameters of the cavity, m; λ_w — thermal conductivity of the wall material, W/(m · K); α_2, α_3 — inner and outer heat transfer coefficients, W/(m^2 · K).

The coefficient of heat transfer α_2 is found from the expression [6]

$$\alpha_2=\frac{q_w}{t_m-t_w}=-\frac{\lambda(\partial t/\partial r)_w}{t_m-t_w},$$

where $q_w=-\lambda\left(\frac{\partial t}{\partial r}\right)_{r_1}$ — heat-flux density at the wall, W/m^2; t_m — average mass temperature; t_w — cooling water temperature.

The average mass temperature of the slip in the annular cavity is determined in the standard manner [6].

Therefore, knowing the temperature and velocity distributions over the cross section of the cavity, the inner heat-transfer coefficient α_2 can be calculated.

The outer heat-transfer coefficient α_3 depends on the properties of the cooling water and the dimensions of the case and is given by the Forchheimer relation:

$$\alpha_3=\frac{2\lambda_b}{d_3\ln(4d_4/d_3)},$$

where d_3, d_4 — diameters of the annular case; λ_b — thermal conductivity of the cooling liquid.

The system of Eqs. (2) – (11) is solved using dimensionless variables. The coordinates z, r are divided by the radius r_1, the velocity components u, v by u_0, the pressure p by the dynamic head, the temperature by τ_0, and the density, maximum shear stress, specific heat, viscosity, and thermal conductivity by their values at the temperature τ_0.

The system of equations (2) – (6) with the boundary conditions (7) – (11) is solved numerically. The computational region is divided into elementary cells with edges Δz_i, Δr_j. The difference analogs of the equations of motion (2) and energy (4) are obtained using the second-order Crank–Nicholson scheme, and the difference analog of Eq. (3) is ob-

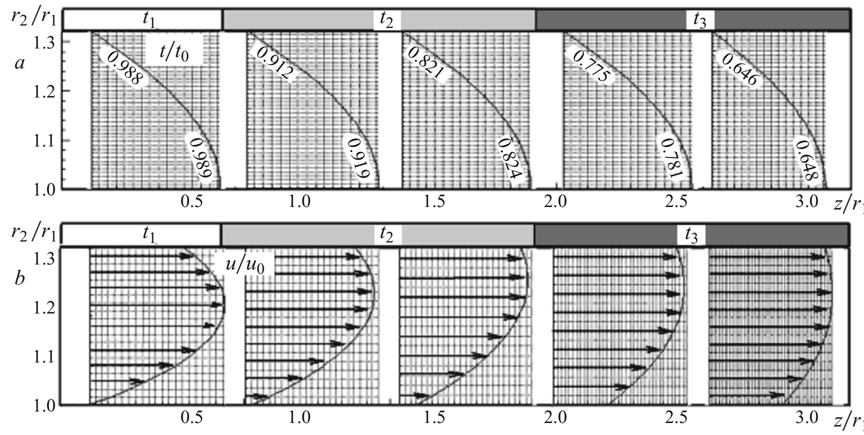

Fig. 3. Temperature (a) and velocity (b) distributions in an annular cavity with $t_0 = 75°C$ and $u_0 = 2$ mm/min.

tained by a two-layer second-order scheme [7]. The pressure gradient is determined by the method of splitting [7] from the flow conservation condition (5).

The problem of heat transfer in laminar flow of a Newtonian liquid in a circular tube with a constant wall temperature was solved in order to verify the numerical method. The variation of the Nusselt number along the tube was obtained in the calculations: it decreases monotonically, goes to a constant value Nu = 3.48, and is identical in accuracy to the analytical solution [6].

DISCUSSION

The calculations were performed for three values of the annular layer $r_2/r_1 = 1.32$, 1.4, and 1.5. The entry slip velocities were $u_0 = 1.0$ mm/min and $u_0 = 2.0$ mm/min. The temperatures of the cooling water in the hot, warm, and cold loops were $t_i = 73$, 59, and 45°C, respectively.

Figure 3 shows the distributions of the velocity u and temperature t in different sections of the annular cavity $r_2/r_1 = 1.32$ with casting velocity 2.0 mm/min. The intensity of the radial heat transfer, arising as a result of heat transfer at the outer wall, increases considerably the heat transfer from the hot slip to the cooling liquid. At the distance $z/r_1 = 0.75$ it is evident that the gradient of the temperature head has a quality effect on the coefficient of heat emission, just as the flow velocity gradient has on the slippage. Since the effect of the pressure gradient is the same over the cross section of the annular channel, the corresponding changes of the viscosity and maximum shear stress on the walls give rise to different values of the velocity on the cavity walls. The slip layers move relative to one another with different velocity, i.e., motion and heat transfer occur in the laminar regime.

The temperature distribution is uniform over the cross section of the annular cavity (see Fig. 3a). This is explained by the fact that because of the high thermal conductivity of beryllium oxide conduction plays the predominate role in heat transfer. Such a temperature distribution imparts uniformity to the rheological properties of the slip. The aggregate state of the slip changes at the horizontal section $z/r_1 = 2.0$. The heat released dissipates rapidly because of the high thermal conductivity of beryllium oxide. In the cold-loop zone the motion and heat transfer of the slip occur in the solid-plastic state.

Thus, the uniform distributions of the temperature and rheological properties at all stages of molding preclude the formation of pits and porosity inside the ceramic as the slip solidifies.

An increase of the thickness of the annular gap $r_2/r_1 = 1.5$ results in a higher mass flow and heat content of the slip as well as in nonuniform temperature and velocity distributions over the cross section of the annular cavity (Fig. 4). The cooling rate of the slip decreases, and solidification starts later at the distance $z/r_1 = 2.2$ than in the case $r_2/r_1 = 1.32$. The temperature nonuniformity over the cross section of the annular cavity results in nonuniformities in the distribution of the rheological properties of the slip. The phase transition starts at the outer wall. The heat of solidification of the slip increases even more the nonuniformity of the distributions of the temperature and, correspondingly, the rheological properties. This can cause the slip to shrink nonuniformly during solidification and internal shrinkage pits and porosity to form at the ceramic formation stage.

Thus, nonuniformity of the temperature and, correspondingly, rheological properties of the slip can result in the formation of a defect during casting of ceramic parts. This is confirmed by the experimental data [2].

Figure 5 shows the variation of the density of thermoplastic slip (a) and the heat flux density on the outer wall (b) along an annular channel. The density variation in the hot loop is small (Fig. 5a). The density of the thermoplastic slip increases as the slip cools; the relative density changes are in the range 1 – 1.01 in the liquid state and 1 – 1.03 in the solid-plastic state. The computed data are in agreement with the experimental results [2, 3].

The variation of the heat flux density at the wall of the molding cavity is the determining factor in controlling the temperature regime of the method of hot casting of ceramic (Fig. 5b). The heat flux density at the outer wall increases sharply in the initial part of the warm and cold loops and decreases monotonically with decreasing slip temperature

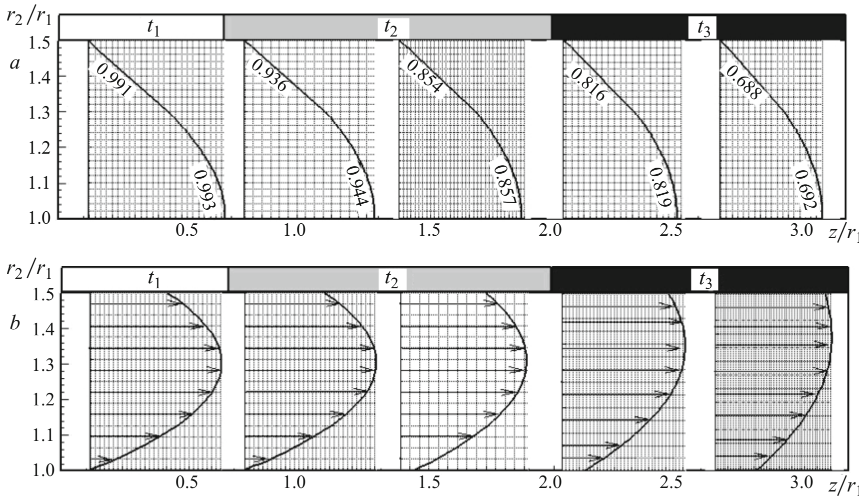

Fig. 4. Temperature (a) and velocity (b) distributions in an annular cavity with $t_0 = 75°C$ and $u_0 = 2$ mm/min.

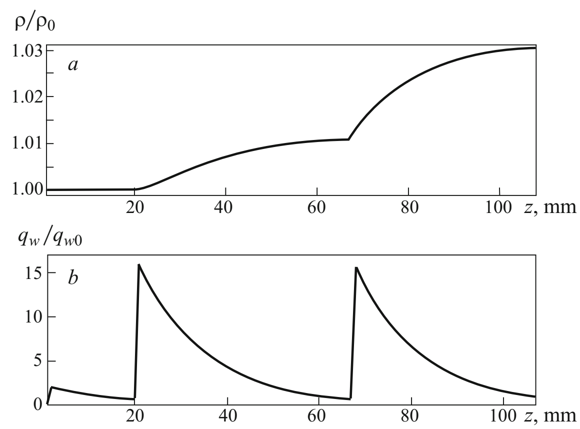

Fig. 5. Distribution of the slip (a) and heat flux (b) densities at the wall with $t_0 = 75°C$ and $u_0 = 2$ mm/min.

(Fig. 5b). The zig-zag variation of the heat flux density is explained by heat emission along the outer wall of the molding cavity due to heat removal in the hot, warm, and cold loops of the annular tube.

The next regime was calculated for annular cavity size $r_2/r_1 = 1.4$ and flow velocity $u_0 = 1.0$ mm/min. Compared with the two preceding regimes, in this case solidification starts at the distance $z/r_1 = 1.9$ and the phase transition front occupies a horizontal section. As the computational results show, this thermal regime also gives uniform rheological properties of the slip at all stages of molding, determined by the temperature field, and a ceramic part of good quality.

CONCLUSIONS

The following basic conclusions can be drawn from the results of this research.

The mathematical model of ceramic molding by hot casting includes a system of equations of motion of a non-Newtonian liquid, continuity, and energy taking account of heat release accompanying a change of the aggregate state as the thermoplastic slip cools in the molding cavity.

The velocity, temperature, and rheological properties distributions showing the internal structure of the slip in the molding cavity were obtained in the calculations. The computational data describe the entire stage of molding of a ceramic taking account of the change in the aggregate state and show that the uniformity of the temperature field over the cross section of the cavity with shrinkage of the thermoplastic slip plays the main role.

The slip density increases as the aggregate state changes. The calculation explains the mechanism of the compensation of the volume changes during molding of the ceramic by hot casting and is in satisfactory agreement with experiment.

The results of this study show that the mathematical model of the process is an effective tool for controlling the method of hot casting ceramic with the optimal conditions determined as a function of the design molds and technological parameters of the molding of thermoplastic slips.

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